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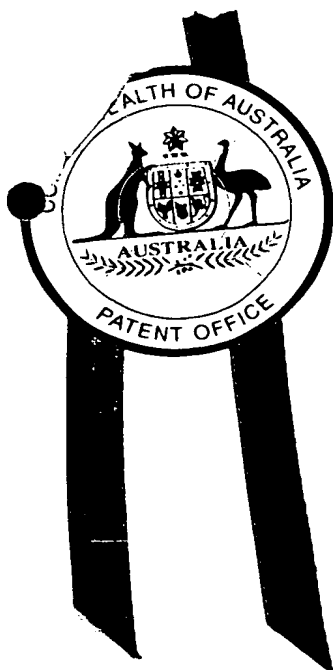
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I, LEANNE MYNOTT, TEAM LEADER EXAMINATION SUPPORT AND SALES hereby certify that annexed is a true copy of the Provisional specification in connection with Application No. PP 6900 for a patent by THE UNIVERSITY OF MELBOURNE filed on 02 November 1998.



WITNESS my hand this  
Fourteenth day of December 1999

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**A U S T R A L I A**

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**PROVISIONAL SPECIFICATION**

for the invention entitled:

"Phase determination of a radiation wave field"

The invention is described in the following statement:

## PHASE DETERMINATION OF A RADIATION WAVE FIELD

This invention relates to the determination of phase of a radiation wave field. More particularly the invention relates to a method in which both phase and intensity can be  
5 quantitatively determined. The invention also relates to a range of applications in which phase and intensity determination can be used. As used in this specification the term "radiation wave field" is intended to include all forms of radiation that propagates in a wave like manner including but not limited to examples such as X-rays, visible light and electrons.

10 There are a range of techniques for the direct measurement of intensity over a plane or surface in a radiation wave field. Phase is typically measured using an interference technique with some form of reference beam. The present invention provides a non-interferometric technique for the measurement of phase. In combination with a direct measurement of intensity a measurement of phase allows the phase and intensity at any other plane in the  
15 radiation wave field to be determined using known techniques.

In accordance with a first aspect of the invention there is provided a method of determining the phase of a radiation wave field including the steps of

- (a) producing a measure of the rate of change of intensity in the direction of radiation  
20 propagation over a selected ellipsoidal surface extending across the direction of propagation;
- (b) producing a measure of intensity over said selected surface;
- (c) transforming said measure of rate of change of intensity to produce a corresponding first fourier domain representation;
- (d) selectively suppressing first higher frequencies of the first fourier domain  
25 representation in the measure of rate of change of intensity to produce a first modified fourier domain representation;
- (e) transforming said first modified fourier domain representation to the spatial domain to produce a first modified spatial domain representation;
- (f) applying a correction based on said measure of intensity over said selected plane to  
30 said first modified spatial domain representation;

- (g) transforming said first modified spatial domain representation to produce a corresponding second fourier domain representation;
  - (h) selectively suppressing second higher frequencies of the second fourier domain representation to produce a second modified fourier domain representation; and
  - 5 (i) transforming said second modified fourier domain representation to the spatial domain
- 
- to produce a measure of phase across said selected plane.

As used in this specification the term ellipsoidal surface is intended to include specifically planar, spherical and cylindrical surfaces.

10

Preferably, the method involves separately performing the steps (c) to (i) for intensity measurements in two mutually orthogonal directions across the planes to produce respective components of the phase and performing the additional step of combining the respective components to produce the measure of phase.

15

In a preferred form of the invention, the first selected higher frequencies are the same as the second selected higher frequencies and the same filter is used for each of the two steps. The filter is preferably selected according to the noise level in the intensity measurements.

20 Preferably, the measure of the rate of change of intensity and intensity distribution over the selected surface are produced from measurements of the intensity distribution over at least two ellipsoidal surfaces extending across the direction of propagation of the radiation and spaced apart along the direction of propagation of the radiation.

25 The selected surface for which measurements of intensity and rate of change of intensity are produced is preferably located between two of the spaced apart surfaces over which intensity distribution is measured.

In the preferred form of the invention the selected surface and spaced apart surfaces are  
30 planar. It is further preferred that the planes are generally perpendicular to the average

direction of propagation of the radiation.

The measure of rate of change of intensity is preferably multiplied by the average wave number of the radiation before transformation into the fourier domain to produce said first

##### 5 fourier domain representation.

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In a cartesian co-ordinate system where the z direction is the direction of propagation of the radiation, the preferred filters are of the form

$$F_x = \frac{(k_x^2 + k_y^2) k_x}{(k_x^2 + k_y^2)^2 + \infty k_x^2}$$

$$F_y = \frac{(k_x^2 + k_y^2)^2 k_y}{(k_x^2 + k_y^2)^2 + \infty k_y^2}$$

10 where

$k_x, k_y$  are the Fourier variables conjugate to  $x, y$  and

$\infty$  is a constant determined by noise in the intensity measurements,  $\infty = 0$  for a no noise case.

The method of this invention thus provides for the quantitative and decoupled determination  
 15 of phase and intensity of a radiation wave field at any surface across the direction of propagation of the radiation. From this phase and intensity determination it is possible to calculate the phase and intensity at any other surface along the direction of propagation. Accordingly, the method provides the basis for a number of measurement techniques.

20 In a further aspect of the invention there is provided a method of imaging an object including the steps of placing the object in a radiation wave field, measuring the intensity distribution of the radiation wave field over a first ellipsoidal surface extending across the direction of propagation of the radiation and spaced from the object on the side remote from the incident radiation, measuring the intensity distribution of the radiation over a second ellipsoidal

surface spaced apart from the first surface or changing the source to object distance to measure a second intensity distribution over first surface and using said first and second intensity distributions to produce a measure of intensity and rate of change of intensity in the direction of radiation propagation over a selected ellipsoidal surface extending across the  
 5 direction of propagation and determining the phase of the radiation wave field according to the above described method.

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The radiation used to irradiate the object can be a planar wave field or spherical wave field or an arbitrary wave field. If it is desired to reproduce the phase in the object plane the phase  
 10 wave field determined by the above method is back propagated and the wave field used to irradiate is subtracted.

It is possible in some applications to use a zero object to image plane distance corresponding to contact-imaging with zero propagation distance.  
 15

If desired the object can be reconstructed in the object plane by back propagating the intensity and quantitative phase information to numerically reconstruct an image of the actual object phase and intensity structure.

20 In other forms of the method more than two image plane intensity distribution measurements can be made to obtain a better estimate of the rate of change of intensity or intensity derivative. In this case one or both of the source to object or object to image plane distances is changed and another intensity distribution measurement is made. The procedure is repeated until the desired number of measurements is made. The measurements provide data to which  
 25 a function can be fitted for the determination of rate of change of intensity in the above described method of phase determination.

The method of imaging an object has particular application to point projection microscopy using X-rays, visible light or electrons.  
 30

In another aspect this invention provides a method of phase amplitude imaging including the steps of

irradiating an object with a radiation wave field;

focussing radiation from the object through an imaging system to an ellipsoidal

5 imaging surface extending across the direction of propagation of the radiation from the object;

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measuring a first intensity distribution of radiation over said surface at a first focus of the imaging system;

introducing a change in focus of the image on said surface through the imaging system;

10 measuring a second intensity distribution over said surface; and

using said first and second intensity distributions to produce a measure of intensity and rate of change on intensity in the direction of radiation propagation over a selected ellipsoidal surface extending across the direction of propagation and determining the phase of the radiation wave field according to the above described method.

15

Preferably, the numerical aperture of the irradiating wave field is smaller than the numerical aperture of the imaging system.

Preferably, the ellipsoidal imaging surface is a detector. The detector is of any suitable form,  
20 such as for example a CCD camera.

Preferably the first focus corresponds to an in focus image at the surface and the changed focus to a slightly defocussed image. Either negative or positive defocus may be used. The defocus is preferably small so that degradation in spatial resolution is minimised. In some  
25 applications more than two images may be obtained to obtain a better estimate of the rate of change of intensity.

In a preferred application the method is used for quantitative phase amplitude microscopy. In this case the imaging system is a magnification system.

30

In the preferred form of the invention the ellipsoidal surface is preferably planar.

The invention will now be further described by way of example only, with reference to the drawings in which:

5        Figure 1 is a schematic illustration of an arrangement for determination of phase

where an object is illuminated with plane wave radiation and point-source radiation;

Figure 2 is a flow chart showing an implementation of the method of phase determination in accordance with an embodiment of this invention;

Figures 3 (a) to (f) are simulated images illustrating phase determination for plane-  
10 wave illumination;

Figures 4 (a) to (m) are a series of images illustrating phase determination and back propagation to another image plane;

Figure 5 is a schematic representation of an arrangement for point projection microscopy using the method of this invention; and

15        Figure 6 is a schematic illustration of an arrangement for quantitative phase amplitude microscopy using the method of this invention.

#### Example 1    Phase determination

20 Figure 1 shows the phase determination where an object is illuminated by plane-wave radiation or point source radiation.

At each point in space, an optical beam possesses two properties: *intensity* and *phase*. Intensity is a measure of the amount of energy flowing through each point, while phase is a  
25 measure of the direction of the energy flow.

Intensity may be measured directly, for example by recording an image on film. Phase is typically measured using interference with a "reference beam". In contrast the present method gives a non-interferometric method for measuring phase.



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Intensity is measured over two parallel planes A, B shown in Figure 1.

The present invention determines phase by providing a solution to the transport-of-intensity equation:

5

$$(1) \quad \nabla_{\perp} \cdot (I \nabla_{\perp} \phi) = -k \frac{\partial I}{\partial z}$$

where  $I$  is the intensity in the plane, the gradient operator in the plane is denoted  $\nabla_{\perp}$ ,  $k$  is the wave number of the radiation, and  $\partial I / \partial z$  is the intensity derivative. Note that  $\partial I / \partial z$  is estimated from the difference of the measurements in the planes A & B shown in Figure 1, 10 while the intensity  $I$  is given by the average.

A solution to equation (1) can be written

$$(2) \quad \phi \approx -k \nabla_{\perp}^{-2} [\nabla_{\perp} \cdot (I^{-1} \nabla_{\perp} \partial_z I)],$$

15 In order to implement a practical solution to equation (2), the following formulae are required:

$$(3) \quad \nabla_{\perp}^{-2} = -F^{-1} k_r^{-2} F, \quad \frac{\partial}{\partial x} = iF^{-1} k_x F, \quad \frac{\partial}{\partial y} = iF^{-1} k_y F.$$

Here,  $F$  denotes Fourier transformation,  $F^{-1}$  denotes inverse Fourier transformation,  $(k_x, k_y)$  are the Fourier variables conjugate to  $(x, y)$ , and

$$k_r^2 = k_x^2 + k_y^2$$

Equations (3) can be used to rewrite equation (2) in the form

5

$$(4) \quad \phi = \phi^{(x)} + \phi^{(y)}, \quad \begin{cases} \phi^{(x)} = F^{-1} k_r^{-2} k_x F I^{-1} F^{-1} k_x k_r^{-2} F \left[ k \frac{\partial I}{\partial z} \right] \\ \phi^{(y)} = F^{-1} k_r^{-2} k_y F I^{-1} F^{-1} k_y k_r^{-2} F \left[ k \frac{\partial I}{\partial z} \right] \end{cases}$$

In practice division by intensity is only performed if that intensity is greater than a certain threshold value (eg. 0.1% of the maximum value);

- 10 Division by  $k_r$  does not take place at the point  $k_r = 0$  of Fourier space; instead multiplication by zero takes place at this point. This amounts to taking the Cauchy principal value of the integral operator  $\nabla^{-2}$ . The phase-retrieval method can be represented by the flowchart shown in Figure 2.
- 15 Figures 3 (a) to (f) shows an example corresponding to the planar illumination scenario given in Figure 1. The example shows the action of this algorithm on simulated noise-free data. Diffraction patterns are calculated using the "angular-spectrum" formalism, an orthodox procedure.
- 20 Dimensions of all images are 1.00 cm square = 256 x 256 pixels. The wavelength of the light was taken to be 632.8 nm, with defocus distance  $\pm 2$  mm. The intensity in the plane  $z = 0$ , which varies from 0 to 1 in arbitrary units, is shown in Figure 3(a). Within the area

of nonzero illumination, the minimum intensity was 30% of the maximum intensity. (The black border around the edge of the intensity image corresponds to zero intensity.) The input phase, which varies from 0 to  $\pi$ , is shown in Figure 3(b).

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- 5 The negatively and positively displaced images are given in Figures 3(c) and (d) respectively, and have respective maximum intensities of 1.60 and 1.75 arbitrary units; the propagation-induced phase contrast is clearly visible in each of these images. The two defocused images are subtracted to form the intensity derivative, which is given in Figure 3(e).
- 10 Images shown in Figures 3(a) and (e) were then processed according to a computer implementation of the method shown in Figure 2, in order to yield the recovered phase map given in Figure 3(f). Note that Figures 3(b) and (f) are plotted on the same greyscale levels, indicating that the recovered phase is quantitatively correct.
- 15 Figures 4 (a) to (h) shows a series of simulated images illustrating phase determination and subsequent back-propagation to another image plane. All images are 256 pixels x 256 pixels = 1 cm x 1cm in dimensions, with the radiation wavelength equal to 632.8 nm. The intensity and phase of the radiation in a given plane are shown in Figures 4 (a) and (b) respectively. Figures 4(c) through (e) respectively show the propagated intensity at propagation distances
- 20 of 199,200 and 201 mm; note the intermixing of information from Figures 4 (a) and (b) in the intensity measurements of Figures 4 (c), (d) and (e). Using the images of Figures 4 (c), (d) and (e) only, the phase-retrieval algorithm obtained the phase map given in Figure 4(f) for the phase of the propagated field at distance 200 mm. Images of Figures (d) and (f) were used to numerically back-propagate the field back to the initial plane. This gave Figures 4(g)
- 25 and (h) for the back-propagated intensity and phase, respectively. These are in excellent agreement with Figures 4 (a) and (b), thus demonstrating the use of the phase retrieval techniques for the quantitative determination of the amplitude and phase of a field over regions far displaced from those over which intensity measurements are made. Note also that the back-propagation is not restricted to free space; back-propagation can also be effected
- 30 through a known optical system.

### Example 2     Point projection microscopy

As shown in Figure 5 radiation for example X-rays, visible light or electrons from a point source (A) is allowed to propagate through free space to the object (B), located at a distance  
5  $d_{so}$  from the source, passes through the object, and is allowed to propagate a further distance  $d_{od}$  to one of the image planes (C) in which the intensity of the radiation is detected. This detection is performed using a standard device such as a CCD camera, image plate or other device capable of registering and digitising the intensity distribution. One or both of the distances  $d_{so}$  and/or  $d_{od}$  is then changed so as to introduce defocus into the images and the  
10 intensity distribution is measured once again. The case of  $d_{od}=0$  corresponding to contact-imaging with zero propagation distance is included as one possible measurement.

The intensity data is then processed using our phase recovery method, described above to recover the decoupled intensity and phase information in the imaging plane. Appropriate  
15 parameters, such as wavelength, pixel size, and defocus distances are inserted into the algorithm, to yield quantitative information about the magnitude of the phase shift in the image plane can be determined.

In certain cases a reconstruction of the object in the object plane, as opposed to the  
20 downstream diffraction planes  $I_1 \dots I_n$ , is desired. In this case the intensity and quantitative phase information obtained above can be used to back propagate the light field to the object plane, thereby numerically reconstructing an image of the actual object phase and intensity structure. This can be done using standard diffraction code.

25 In some cases it is desirable to take more than two images in order to obtain a better estimate of the intensity derivative  $dI/dz$ , in which case one or both of the distances  $d_{so}$  and/or  $d_{od}$  is altered once again and another image taken, with this procedure repeated until the number of desired images is acquired. A function can then be fitted to this data from which  $dI/dz$  can be computed and used in the phase recovery algorithm in place of the simple subtraction of  
30 two images normally used.

### Example 3      Quantitative phase amplitude microscopy

As shown in Figure 6a the sample is illuminated using a source of white light Köhler illumination (A), commonly found on optical microscopes. The light transmitted through the object (B) is collected by the microscope imaging system (D) and relayed to a CCD camera (E) or other digital imaging device. Three images are collected: an in-focus image,  $I_0$ , and two slightly out of focus images  $I_+$  and  $I_-$ , the defocus having been obtained by means which are not critical to our technique, although we simply adjust the microscope focus knob. The defocus introduced is usually quite small so that degradation in spatial resolution is minimised, although the optimal amount of defocus to use is determined by sample properties and imaging geometry (magnification, numerical apertures, etc.) so it is not possible to give a fixed figure for the best defocus for all cases.

When taking the images the numerical aperture of the condenser is chosen to be less than the numerical aperture of the objective being used. If this is not the case then serious image degradation will occur, although the precise amount by which the condenser and objective numerical apertures is used in the phase determination method described above should differ involves a tradeoff between image fidelity and spatial resolution, with the optimal difference depending on the sample properties and the optics used.

The intensity data obtained to recover the decoupled intensity and phase information in the imaging plane. If appropriate parameters (such as wavelength, pixel size, and defocus distances) are applied, quantitative information about the magnitude of the phase shift in the image plane can be determined.

As in Example 2 for point projection, there may be cases in which it is desirable to take more than two images in order to obtain a better estimate of the intensity derivative  $dI/dz$ . A function can then be fitted to this data from which  $dI/dz$  can be computed and used in the phase determination method in place of the simple subtraction of two image normally used.

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It is also possible to operate this system in a reflection geometry as well to obtain surface topography. The principle of operation is the same, however the optics have to be folded back on themselves to form a reflection geometry - otherwise the process is identical.

5 For certain applications it can also be desirable to filter the light to a particular wavelength,  
although this is not necessary for the described imaging process as it works equally well with  
white light.

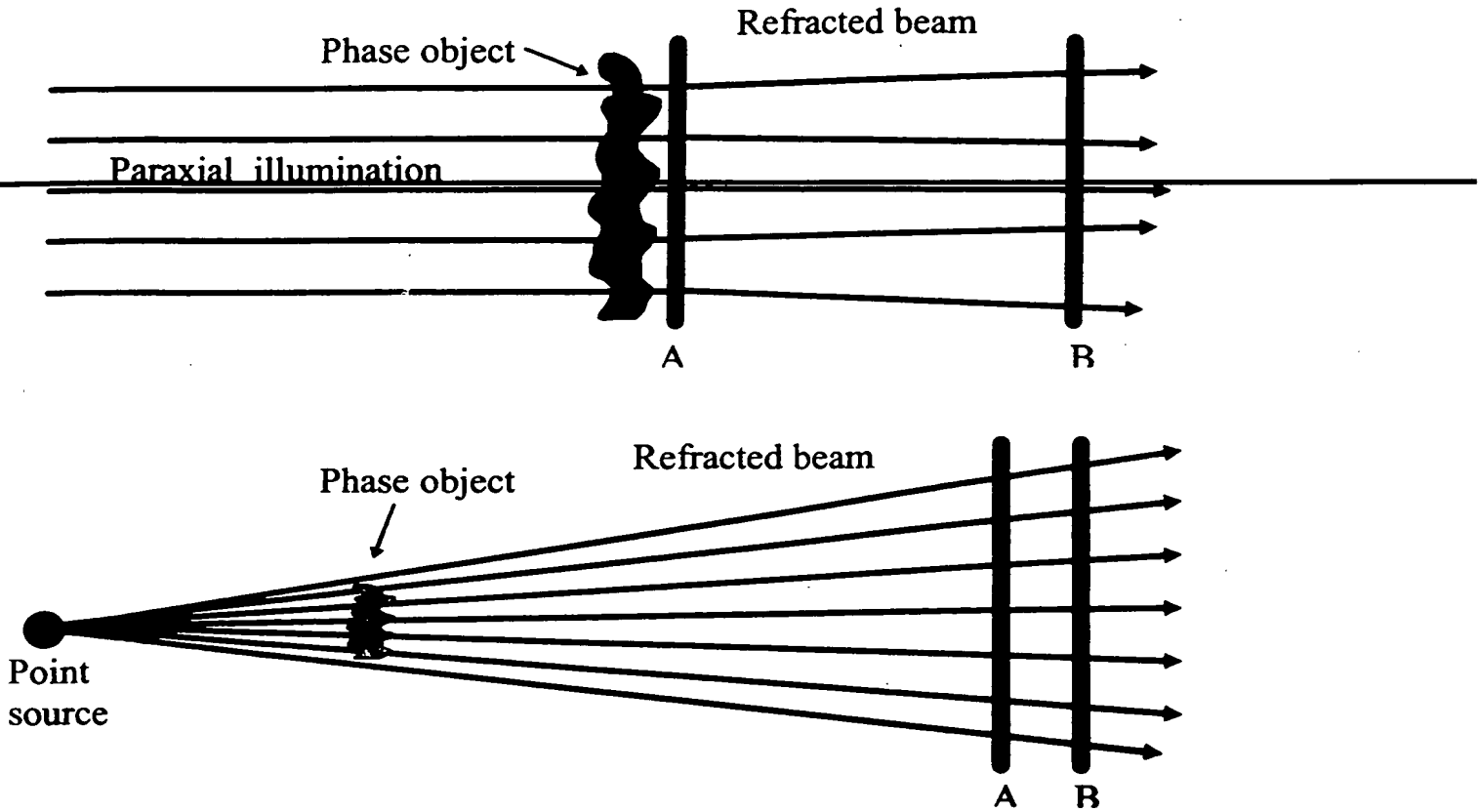
A system operating on the same principles can also be used for tomography.

10

DATED this 2nd day of November, 1998.

15 DAVIES COLLISON CAVE

Patent Attorneys for the Applicant



**Figure 1 : Generic phase-retrieval setup. Plane-wave illumination (upper panel) and point-source illumination (lower panel).**

- Inputs:** (a) Intensity measurements  $\{I_n\}$ , including central intensity  $I_0$ ;  
 (b) noise level  $\sigma$ ; (c)  $\alpha = \alpha(\sigma)$  is a noise-dependent regularisation parameter;  
 (d) noise-dependent filters  $F_x = \frac{k_x(k_x^2 + k_y^2)}{(k_x^2 + k_y^2)^2 + \alpha k_x^2}$  and  $F_y = \frac{k_y(k_x^2 + k_y^2)}{(k_x^2 + k_y^2)^2 + \alpha k_y^2}$ .

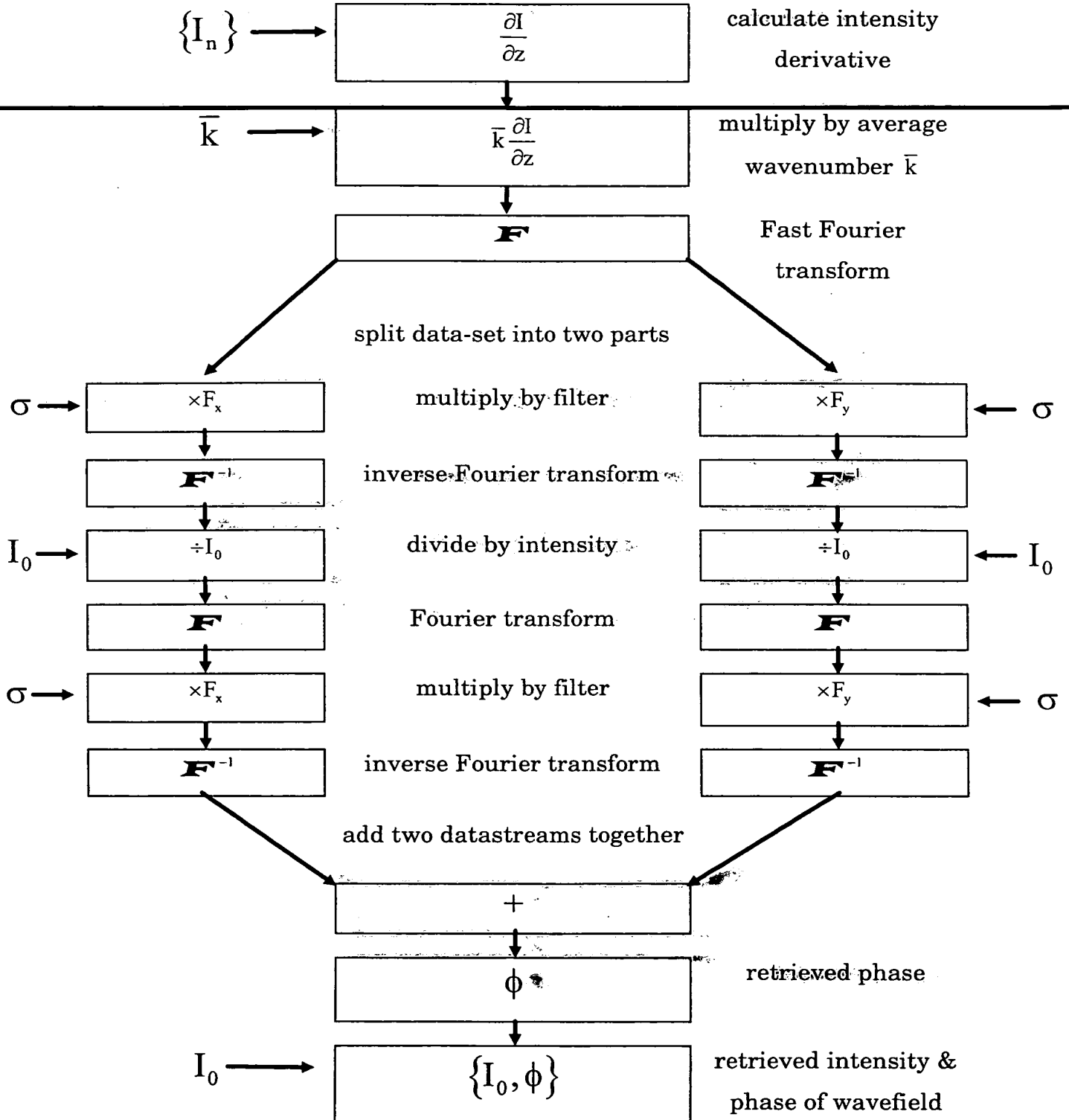
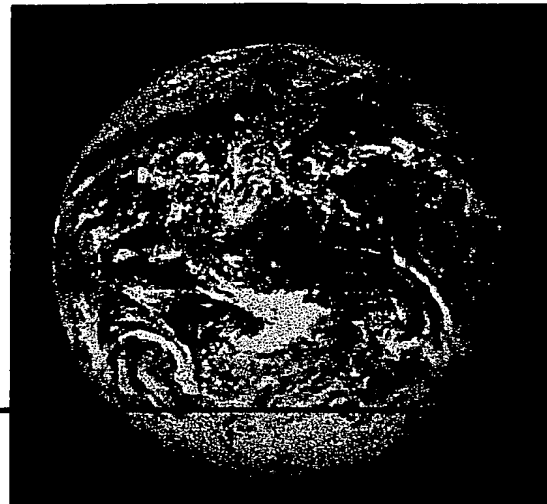


Figure 2

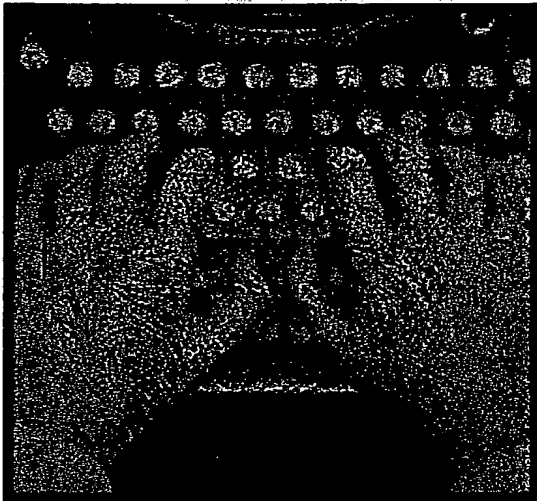




(a) aperture plane intensity



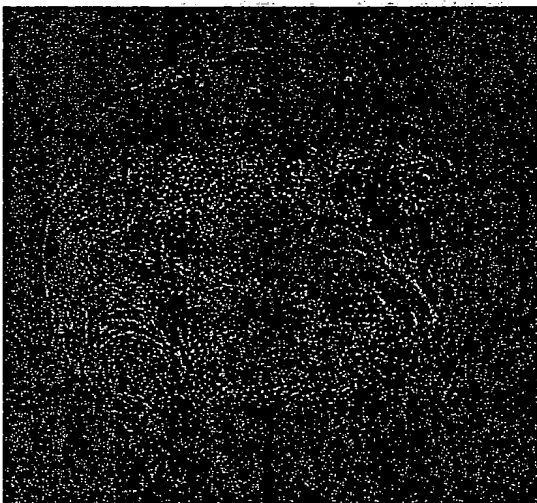
(b) aperture plane phase



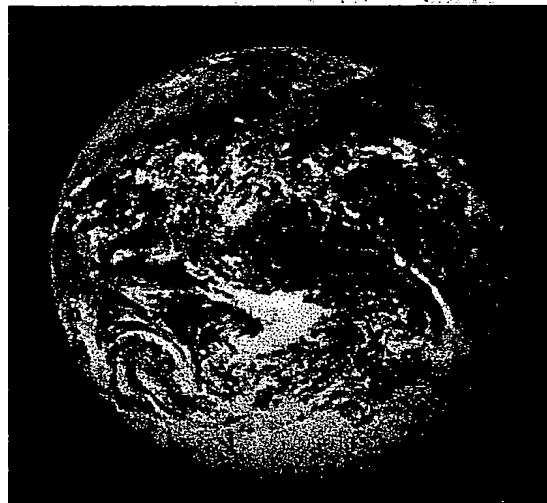
(c) negatively defocused intensity



(d) positively defocused intensity

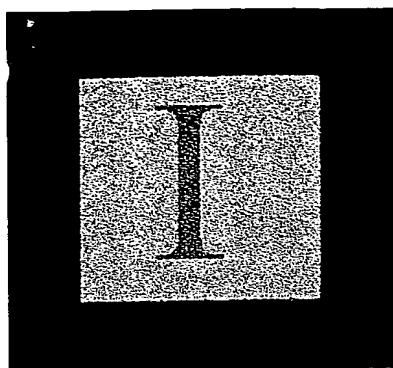


(e) intensity derivative



(f) recovered phase

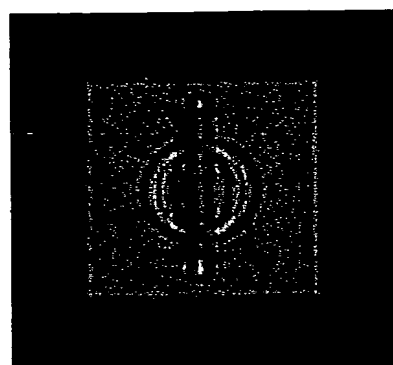
**Figure 3**



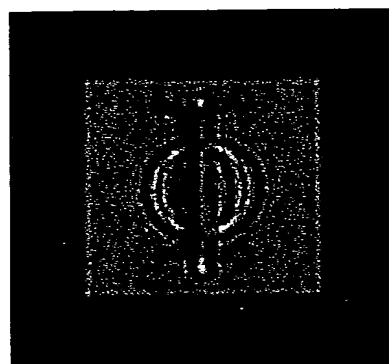
(a) intensity,  $z = 0$  mm



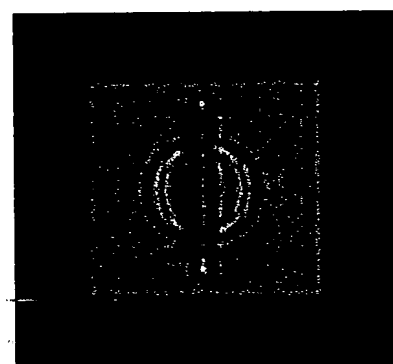
(b) phase,  $z = 0$  mm



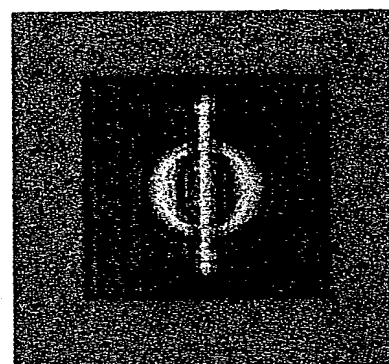
(c) intensity,  $z = 199$  mm



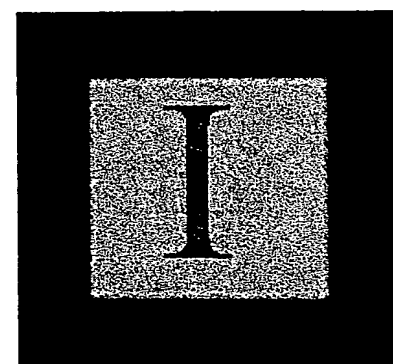
(d) intensity,  $z = 200$  mm



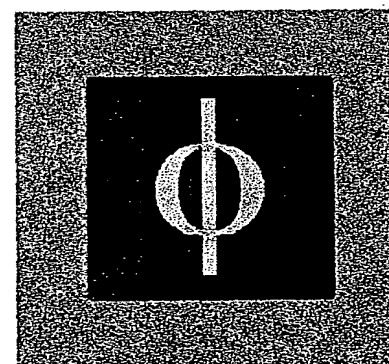
(e) intensity,  $z = 201$  mm



(f) retrieved phase,  $z = 200$  mm

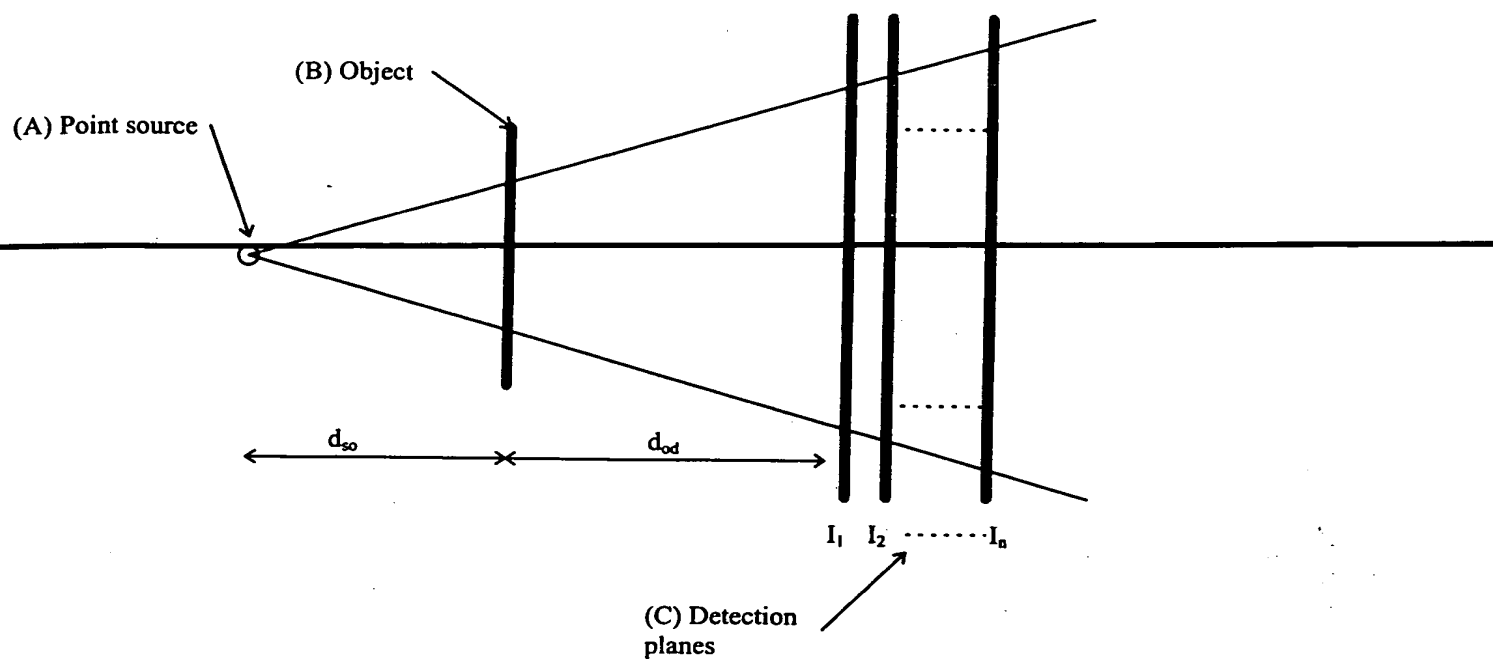


(g) back-propagated intensity,  $z = 0$  mm



(h) back-propagated phase,  $z = 0$  mm

Figure 4

FIGURE 5

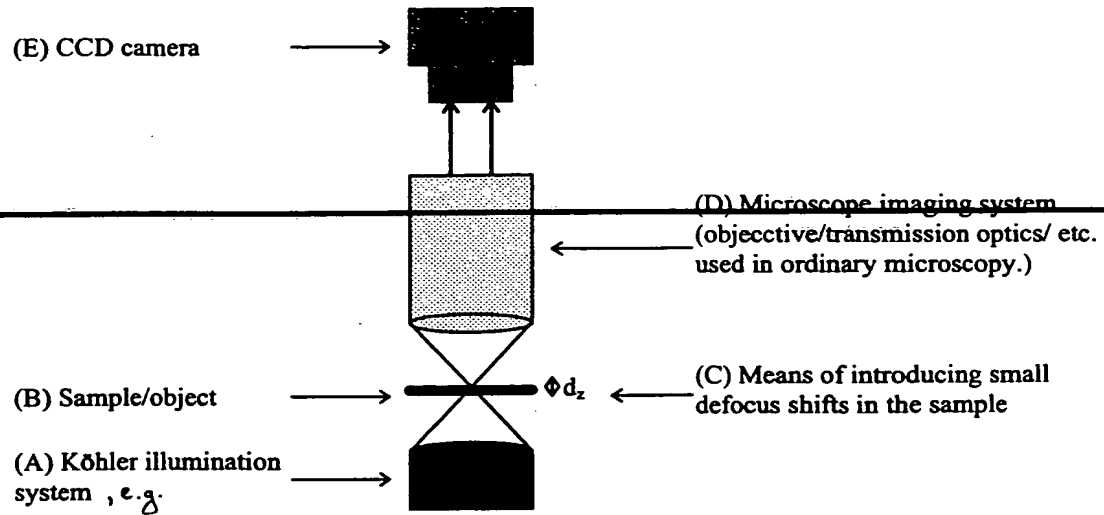


FIGURE 6